**Question 1**

|  |  |
| --- | --- |
| Find the work done in moving a particle along the quadrant circle that has radius 3, going from to in the *x-y* plane through the force field *F* given by:  Note that the force field *F* shown in the diagram is not the field described by parts (A), (B), or (C). |  |

**Solution**

The work done by a force , given in Cartesian coordinates, in moving a particle along a path *C* given by from 0 to is

Express the given quadrant of the circle, radius 3, as:

The components of r in terms of *t*, that is, the parametric equations for C, are

where *t* goes from *t0* =0 to *t1* =1,

and diﬀerentiating these equations yields

The components of *F* in terms of *t* are

Substituting into equation (1) gives

The negative result indicates that the force field resists anticlockwise movement along the quadrant.

**Answer:**

**Question 2**

|  |  |
| --- | --- |
| Use Green's theorem to find the counter-clockwise circulation for the field *V* and the curve *C*: the triangle bounded by the points *O*, *A* and *B*. (Hint: Check your answer using line integral involving 3 separate lines.) |  |

**Solution**

Using the conclusion of Green’s theorem as:

find the counter-clockwise circulation for the field *V* around C.

To compute the line integral, we note that

The curve *C* (broken down into three pieces) is pictured. On *OA* we have

which gives us

On AB we have

which gives us

On BO we have

which gives us

We conclude that

Next we evaluate the double integral and see that we get the same result:

**Answer:**

**Question 3**

In the following cases you are given a 3D velocity vector field *v*. This represents the velocity of a fluid at the position Find the total amount of the fluid that is flowing out of the unit cube ().

**Solution**

For solution question we need to find the flux of the velocity vector field *v* out of the unit cube:

The cube has six sides with , with , with , with , with and with .

On , the outward normal is and

.

On , the outward normal is and

On , the outward normal is and

On , the outward normal is and

On , the outward normal is and

On , the outward normal is and

With some amount of simplification of the integrands being possible since one coordinate variable has a fixed value across the entire cube face.

The six integrals are then

Since the area of each side is one it follows that

**Answer:**

**Question 4**

Calculate the curl of each of the following vector fields (), and prove that

**Solution**

The curl of a vector ﬁeld in Cartesian coordinates is

For our question find:

then

Prove that :

**Answer:**

**Question 5**

Determine the general and particular solution of the second-order differential equation with the initial conditions.

(1)

with

**Solution**

Let’s write the characteristic equation for the left part:

This is a [quadratic equation](http://www.sosmath.com/algebra/algebra.html). its roots are then the general solution of differential equation without right part is

The right part of the equation has a form:

As 0 is not a root of the characteristic equation , then we will find the particular solution in the form:

where

let's substitute in (1):

let's equate coefficients at identical degrees *x:*

from where:

Then

The general solution is

Find and . Use conditions:

substitute in (2) and (3):

The particular solution is

**Answer:** The general solution: the particular solution